ON INNER PRODUCT SPACES OVER ZEROSUMFREE SEMIRINGS

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Abstract: A semiring A is called a zerosumfree semiring if for all $a, b \in A$, $a+b = 0$ implies that $a = 0$ and $b = 0$. Here we have endowed every semimodule over a zerosumfree semiring A with a natural inner product of values in A. A dimension theorem for orthonormal bases of these inner product spaces over zerosumfree semiring is proved. The main results in this paper generalize the corresponding results on the Boolean inner product spaces [5].

1. Introduction and Preliminaries

Linear algebra over distributive lattices, Boolean algebras, incline algebras and semirings etc. has a long history, as well as holds an important position in the modern theory of linear algebra due to their applications to computer science, optimizations and theoretical physics [3,7]. Several authors have studied invertibility of matrices and linear operators on the vector spaces over such algebras [2,6,9,10,11,12,13,15], whereas some others are developing the theory of vector spaces over such algebras [14,16,17]. In 2009, Gudder and Latremoliere [5] developed inner product spaces over a Boolean algebra. Gudder and Latermoliere [5] generalized some results on power of matrices over the two element Boolean algebra $\{0, 1\}$ [4] to arbitrary Boolean algebras. In this paper following the idea of Gudder and Latermoliere, we have developed inner product spaces on zerosumfree semirings. Since every Boolean algebra is a zerosumfree semiring, our results generalize the corresponding results on the Boolean inner product spaces. It is