MODIFIED SADDLEPOINT DENSITY ESTIMATES

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Dedicated to Prof. A.M. Mathai on his 80th birth anniversary

Abstract: This paper proposes a density estimation technique whereby a moment-based adjustment is applied to the saddlepoint approximation as determined from the empirical cumulant-generating function associated with a given set of observations. When two variables are involved, the product of saddlepoint density estimates of the marginal distributions is adjusted by means of a bivariate polynomial. Unlike kernel density estimates, the modified saddlepoint density estimates have simple functional representations that readily lend themselves to algebraic manipulations. Since the proposed methodology relies essentially on a determinate number of sample moments, it is particularly well suited for modeling massive data sets. As well, it should lead to improved density estimates in connection with the countless current applications arising in various fields of scientific investigation. For illustrative purposes, the density estimation approach being advocated herein is applied to two univariate and two bivariate data sets.

Keywords: Saddlepoint approximation; density estimation; moments; empirical cumulant-generating function; big data; bivariate density estimate.

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1. Introduction

The saddlepoint density approximation that was introduced by Daniels (1954), as well as its later refinements, may prove inaccurate when applied over the entire support of certain continuous distributions. This deficiency is illustrated in Figure 1.1 where the saddlepoint approximation of the density function of a certain mixture of normal densities previously considered by Huzurbazar (1999) is superimposed on its exact density function. This paper aims at addressing such shortcomings, mainly in the context of density estimation.